Concerning the Solution to the Liar Paradox

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Abstract
In this note, we analyze the Liar Paradox. We re-write the problem in order to properly include it in the body of Science and prove that the problem is an allurement to show the complexity of the human mind in the same sense that the Sorites Paradox is an allurement to show the complexity of the human verbal expression.

Keywords: paradox, liar, Sorites.
Introduction

The Liar was formulated by at most the second century after Christ (see [Paul Vincent Spade, 2005]). When put against The Sorites, it is not great in wealth, but both problems could be told to be means to illustrate some important scientific teachings.

In The Liar, the idea is having one entity (group, person, machine or other) judging the speech of another entity and coming up with a verdict that be universally accepted at least in the scientific métier.

The Liar assumes that the verbal expression of human beings must not be personal, must not represent individuals, but collectivities.

Every person is entitled, however, to verbally say whatever they like about someone else's speech and whatever they say cannot be the main object of any scientific discussion of serious nature if such a verbal expression has to do with usual events of human life and personal impressions.

The Liar may appear in more than one way, but a presentation that copies the original problem cannot be much different from the one that we describe below:

1) Assume that someone has uttered \( p \), \( p \) being:

   \[ I \text{ always lie.} \]

2) Assume that you were listening to them and that you have been given two choices (classical choices):

   a) Writing that their statement is true

   or (exclusively)

   b) Writing that their statement is not true (false or anything else).

3) Assume that a choice has been made by you in 2.

4) Would you then believe that the same subject, if uttering \( q \), \( q \) being

   \[ \text{Would you believe me?} \]

   deserves a

   a) 'No' (anything that be not yes)

   or (exclusively) a

   b) 'Yes' (classical choices)

   as a response from you?
We here analyze The Liar in each one of its steps (four steps, as described in our model) without ever translating the problem into symbols because of the level of reasoning that is involved in it (the own problem, not its solution: We should not put a box, $X$, inside of another box, $Y$, if $X$ is much bigger than $Y$ and cannot be diminished in size without losing its original appearance forever).

Step 1: The statement, as uttered by the subject

First of all, the imagined situation is strange: Someone comes, out of context, and utters that they always lie or always tell the truth or do something else that implies judgment of their honesty, in a universal way, like of their every statement in life, what provides the audience of theirs with unreliable information gain, that is, with no mathematical (we call classical-logic material mathematical material) inferential material, at most with statistical inferential material instead.

If a classical computer (we say computer because anything that may be classified as purely logical has to be passive of programming and we say classical computer because the only accepted answers are yes or no, classical logic answers) is listening to that, no input will ever be made.

Why?

A speech, of a person judging themselves, has got no logical value: A system can never judge itself on any grounds (this is not a valid process or a process that may lead to universal inferences, as the problem seems to demand).

It is obvious, even for a child, that the only person able to emit judgment is that who is out of the system...: If you are inside of a bubble, you cannot judge the nature of it because you do not hold information about all that exists outside of it and judgments may only be made by those dominating all available matters of the same nature, or level, of the attrib (reference to the lingo of Maple\(^1\)) function contained in the utterance, therefore they can only be made by people living outside of the bubble and with no contact with its surface (this situation is similar to that justifying the necessity to see the planet from the outer space, rather than from inside of it or from over it, in order to judge, in a universal way, its shape).

The first utterance is then empty in logical value (classical) and scientifically unacceptable (Science is not a self-referential place, basically). To make that statement become scientific, we would need to replace 'I' with a 'cold' subject's name, for instance (something like Mary always lies, even if it is the own Mary speaking about herself, just because we need the detachment, quality expected and needed in scientific discourse, to consider including this problem in the body of Science). To

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\(^1\) See http://www.maplesoft.com/products/maple/
make of that statement something logically useful (a logical system is a *machine* through which we can produce inferences), we need a third entity. What may then be introduced is, for instance, a tool, say a liar detector (see [John A. Podlesny and David C. Raskin, 1978]), to tell us the actual information content (mathematical) of that assertion.

The human world is obviously not logical, only the machine world is... .

This way, it is nonsensical to even think of that as a logical statement or as a proposition. That is at most an incomplete logical proposition. It will become a logical entry, or proposition, that is, a unit that Philosophy (we have written that Philosophy should mean scientific discussion of whatever is logical in the human actions on earth and may be expressed, perfectly well, through language) can deal with, after we aggregate the judgment made by the liar detector to the already existing statement. The proposition will then be the revised $p$, now with a compound sentence (see [Michael Genesereth, 2011]): (Mary always lies, machine has returned $x$ ($x$ replaces *true* or (exclusively) *false*).

Of course it is assumed that there are only two possibilities for a liar detector, classical ones, and that is true in reality, so that there is no room for any other speculation of delusional nature (both true or false, for instance). Notwithstanding, there are occasions in which the liar detector will not be able to judge. In those occasions, we could ask the person the same question in an alternative way or use other means to explore the question, but we would have to always be able to get either a *yes* or a *no*, and exclusively one of these two options, as a result of the application of the chosen method in order to be able to progress to the next step of the problem. We also need to apply a method that be accepted by Science here.

**Step 2: You are part of the audience and must now write *true* or *false***

Indeed, you were listening...

Now we must manufacture a situation in which you judge, in real life, if the utterance is true or false, that is, a situation in which you bother about what they utter and still dare knowing more than the utterer about what they themselves state regarding themselves... (it is clearly the case that the simple thought that you can do that makes of you a megalomaniac person, for you cannot know more than the own utterer about themselves, but we keep on going...).

A balanced person would state, in this situation, perhaps like Christ would: You said it, now tell me.

However, that utterance would still have mathematical and real-life content zero, independently.
Science should be about observing; identifying, describing, investigating or theoretically explaining phenomena (see [Freedictionary, 2011]). What are the phenomena we here refer to? A person uttering a sentence. Are we observing them? Yes, we are. Are we identifying a new phenomenon here? No, we are not. Are we describing a new phenomenon here? No, we are not. Are we investigating further a known phenomenon? Perhaps. Are we theoretically explaining some new phenomenon? No, we are not.

The most suitable answer would be that we are investigating further a known phenomenon, which is how the logical/machine truth connects to the human speech.

The problem is that we do not have enough elements to perform this investigation inside of Science.

It is obviously not an investigation that may be run only in Philosophy because we can only learn the degree of truth involved in the utterance investigating the own mind of the person to, first of all, be sure that they believe what they say, like that they are not lying to us knowing that they are doing so.

We do believe that a human being may lie in each and every utterance of theirs in what could be told to be universal way.

Analyzing the truth involved in that utterance therefore means analyzing the connection of that oral expression to the mind of the utterer, first of all, what can only be done through tools that go beyond Philosophy.

On the other hand, the individual may deeply believe that they have never told a lie and hold some memory problem, therefore not actually remember that particular occasion, when they were five years old, for instance, and lied to their mother about something for at least a few hours.

In this case, to investigate the amount of truth contained in that utterance, we should go also for the facts of the life of the individual, what would then mean having a report that we could believe, in a scientific level, about their every utterance in life, just because the own individual can be universally told to be incompetent to judge themselves regarding their own utterances and the degree of truth contained in them.

Oh, well, now we have to involve at least investigative services to be able to provide some scientific reliable theory on the connection between facts and their utterance.

It definitely looks like this is absolutely impossible even if only one individual is considered.

There is therefore no Science in The Liar, just allurement.
However, we can go up to the level of the connection between utterer’s mind and utterer's speech, so that we would be analyzing how much coherence of the individual’s assertions and their own thoughts exists when the individual states that particular sentence to someone else and our truth starts being attached to that restriction, that particular new universe that we have created inside of Science.

That is fine because Science is founded on artificially restricted observational universes.

In case we can examine this problem inside of Science, we must know that the problem that we will analyze is the connection between the mind and the human speech, not the connection between the logical truths and the human speech.

We then need to include a scientific tool of analysis of the human mind in the solution to this problem.

Suppose that we have done that.

The next step would be adding the information from the liar detector (or from the alternative tool in case there be need), which is machine-friendly, to form the needed proposition, that containing the result of the analysis (yes or no) of the main statement of the problem.

Now, it is us repeating the information attained via machine, what just puts sense in all (machines-talk, same pattern of reasoning, inference, and decision, that is, the whole problem has been put into a classical logic perspective and, therefore, has been made passive of judgment and presentation of solution there, as explained in our work from 2008).

Step 3 has already been taken now since we have replaced the ‘you’ there with the scientific tool and ‘a choice’ with a declaration (from the scientific tool or formed from its results).

**Step 4: Would you believe them? Please write yes or no**

Notice that the title of this section ads information: Logical entities must be things that we can write about, never things that we can only think about, that is, to deserve being considered logical information, the thing must be passive of being told in writing, not only in oral discourse, to others.

Expressing this sort of opinion is, unfortunately, a logically useless action. The person doing this may, as well, lie or tell the truth.

Therefore, the question does not make any logical sense and any logical speaker would say: Well, based on the liar detector test, yes (or no).
The logical speaker would exclude themselves from the environment that the problem wishes for belonging to, the logical environment (a human being, if normal, cannot, possibly, entirely belong to the logical world, so that the exclusion is necessary).

Basically, The Liar is not fun: There is no paradox once more and the level of reasoning that it reaches is far lower than the level that The Sorites reaches.

It cannot be a paradox because there is obviously no conflict in life: A person would never waste their time, if ever depending on such judgments (even if the judgments were necessary to save their own existence).

Suppose that a person, who we will call X, be pointing a gun at another person's face (Y), and X state that if Y solves the paradox, they will save their life.

What would Y do, if ever acting logically?

Y would challenge X by giving them another paradox.

Why bothering?

Playing scientifically with illogical speakers?

You do not waste your time, you do not try; you distract them from the fixed idea, and that has to be the only logical choice available!

Your life is at stake, you will not go probability: You can only have a chance to win if you make them be as confused and busy as they try to make you be while you think of ways of getting out of the gun's nose.

Well, once more, as we did with The Sorites, if you think that The Liar is a paradox, point what word comes after paradox (paradox of language? Paradox of logic?).

Now we consider each one of the just-mentioned possibilities:

a) Paradox of language: In language, there is allowance for anything to happen, even for the presenter to ask the question and the victim to leave the place with an uh! (the victim is not obliged to play the game!), so that there cannot, ever, be a single paradox in Language. Language is a tool, not a logical place: It is a tool for entertainment, communication, and expression.

b) Paradox of logic (of the chosen logical system): Apparently, it is here that they place it. If they are right when doing that, then there must be at least one possible couple of inferences, contradictory inferences, passive of deduction from the same propositions and evaluations. Notwithstanding, both input and output of the problem are classical, the input having demanded, once more, fitting a larger object into a smaller one. Such a translation can only be a forced move. What is required for that forced move to be plausible and scientifically acceptable is the use of logical
tools, trivially. We have then introduced the liar detector, which will change confused linguistic speech (broad sector, involving emotions, logical decisions of any level, even impairment, for a person may not be able to say lying or telling the truth, for instance...) into bivalent one (true, false). For this problem to deserve being called paradox, it is necessary that the classical logic systems involved create confusion, that is, produce two possible results for each valid interpretation. However, once more, it also seems impossible to find paradox in this area if the right elements (or complete) of analysis are used.

Notice that if the machine returns ‘true’ and the person has said ‘I always lie’, then that could mean that they also lied this time, what would then mean that it is not true that they always lie, like, at least when saying the last sentence, they were not lying.

What we then see here is a temporal issue.

Whilst the person is stating this sentence, ‘I always lie’ is still true, but, when they finish stating it, in case the machine return true, then the sentence is not true anymore because they have just spoken the truth, like they have done that for the first time in their life (this according to their memory (or health) at the time, not to be forgotten).

Each time a human being ponders about their character like this, they can only analyze the past, never what is happening at that very moment, just because it is not complete yet, like they are still inside of ‘the bubble’…

If asked again, one second after they finish stating ‘I always lie’, they will then say ‘I almost always lie’, for instance, since now they have a recollection of at least one instant in their lives in which they told the truth…

Notwithstanding, if an individual always lies, it must be true that they will say that they tell the truth always instead, like this just makes sense.

The sentence is also something almost impossible to judge with the determinism demanded by the problem, as pointed out before, since it is not a logical sentence, that is, it is not a sentence that could be part of a logical problem or of any serious exercise of scientific analysis.

As they say, some people believe deeply in a reality that does not exist, for instance. The reality that seems to be a result of hallucinatory processes to others is seen by the person as an absolute truth, so that ‘lying’ and ‘telling the truth’ a sort of merge here, like what the person who invented this problem would like to see is an equality between the human world and the machine world, but such an equality should not exist, like ever.

‘I always lie’ does not imply that ‘I always lie’ is also a lie because, attached to this sentence, there is a human being with emotions and perceptions, not a machine.
However, notice that if ‘I always lie’ implies that ‘I always lie’ is false, then I might have told the truth for the first time in my life when I said ‘I always lie’…, like where is the actual conflict then?

As the human being lives, things change, like things change all the time for the human being, starting with the state of their own body, so that nothing is the same that it was one second ago and one cannot go with the reality of this second to evaluate the second before this one…

‘I always lie’ was true until they finish saying ‘I always lie’, what now has a ‘sort of healed them’, and may even make them always tell the truth from that moment onwards, for instance.

One can easily understand the difference we talk about here if one imagines a person with cancer, for instance.

They are in the stage of learning how to deal with their disease, so that they are speaking to a nurse and they are saying ‘OK, I have cancer’.

Does that mean that they cannot ever stop having it?

Does that mean that they will be lying if some day they come to the nurse and say ‘I do not have cancer’, like can someone turn to them and say ‘but you said that you had cancer at the meeting of 04.05.2012’, for instance?

No, nobody should do that.

So, that is the same situation, we have just changed the contexts involved.

Imagine, putting things in an even clearer manner, that we are the presenter of the problem.

We then say to the person, at that stage: You are lying because this sentence contradicts frontally your other sentence, from the past, therefore YOU HAVE CANCER.

Even in the machines world, unfortunately, for the person who built this problem, things get updated quite frequently.

Maple, for instance, may change the variable x from 0 to 1 if the user of the program writes another ‘attrib’ sign close to x and 1 instead of 0.

Does Maple crash at that stage?

No, it does not.

Maple accepts the update and deals with x replacing it with its new value, 1 (Maple therefore uses time as a coordinate).
As another computer example, consider the lines of instruction \( B := \text{all variables are zero} \). Print \( X \). \( X(B) = f(x) = x+1 \), say. This implies \( f(0) = 1 \) and \( f(0) = -2 \), therefore it implies that \( f \) is not a function.

\[
f(y) = y - 2
\]

Does the fact that \( B \) makes the variables of the ‘system’ be meaningless make \( B \) itself be meaningless? No, it obviously does not. Yet, we could have written \( B \), which is a variable, like this: “The variables are always meaningless” (since 0 is a meaningless parcel in both sums in \( X(B) \)). Notice that transferring things that belong exclusively to the mathematical domain to the broad universe of the human language is as catastrophic as transferring things that belong exclusively to the mathematical domain to the broad universe of the human language, like to whatever is not in the intersection between the universe of Mathematics and the human universe, to the mathematical universe.

Sadly, not even in the machines world, like at least not all the time, ‘I always lie’ implies that ‘I always lie’ is a lie.

Everything comes, in life, for human beings, attached to several other things, say, and one of those things will always be time.

Perhaps one should read our paper on the Cartesian System to understand better what we say here.

‘I always lie’ is at least (‘I always lie’, \( X \) (subject who said that), date \( W \), time \( R \), true or false (for the utterer)), for instance.

In time \( R+1 \), however, ‘I always lie’, which got the value true, say, in the last sentence, will get the value false.

Perhaps things could be even better understood if we said that ‘always’ means up to this moment, for instance. However, some human beings, as they utter the truth for the first time in their life, will add, at their own initiative, ‘but now I am not lying’, for instance.

On the other hand, other individuals may be planning to tell lies from that moment onwards and decide to ‘advertise their intents like that’, ‘I always lie’, yet they have told the truth up to now, what then would have to make us change the meaning of ‘always’ again… .

Basically, neither \( \text{always} \) nor \( \text{lie} \) can be translated into logical lingo. ‘I’ also cannot, as we said right at the beginning of this paper. The entire sentence can never be translated into logical symbols. Whoever created ‘the paradox’ thought like this:

1) \( p := \text{I always lie} \)

2) \( v(p) := 1 \)
3) Therefore ‘I always lie’ is true, therefore ‘I am also lying when I say ‘I always lie’’, therefore ‘I always lie is false’, that is, ~ p.

4) Therefore v(p \land ~ p) = 1, what is absurd (since, in classical logic, each proposition can only have one truth value at a time, that is, it is either the case that p is true, therefore v(p)=1 and v(~p) = 0, or that p is false, therefore v(p) = 0 and v(~p) = 1).

Notice that, to reach their equivocated conclusion, they inadequately translated usual language into logical language, then they stopped the inferential process (let’s call this process 1) to translate what they had in the logic into usual language to produce their new inferences, when they then came back to the process 1 ‘to keep on going’ as if ‘it were all normal and acceptable’. One would think, however, that if you translate from the usual language to the logical language it is already risky enough to come back to the usual language at the end of the inferential process, who would say doing this?

Nobody has ever stopped to actually ask adequate questions here, such as: What theorems are you using, proposer, to make these ‘conversions’ valid in classical logic, please? One must remember that we can only accept a move in Mathematics if the move is made because of an extremely solid mathematical theory, that is, if it is made because of a theorem, lemma, convention, or alike.

Hopefully, this was enough to prove our point.

Now, we would then suggest, to those who are still not believing what we say, that instead of dealing with humans the same way we deal with the machines, with people who are not alive and what they have said, or others, that is, without a time constraint and the chance that it all changes next second, they add at least one coordinate to their ‘point’, say.

Instead of having (‘I always lie’, X), that they now have (‘I always lie’, X, date, time), so that they can see and accept that (‘I always lie’, X, 24.09.2012, 9:18PM) may be true, and so may be (‘I sometimes tell the truth’ or ‘It is not true that I always lie’, X, 24.09.2012, 9:18:02PM).

Some researchers have said that one needs to add something to the sentence, say ‘I always lie, but I am not lying now’, to make it acceptable logically.

We say that we need to add something to the mind of the person who is analyzing it instead so that they accept that things are like this and are ‘just fine’… .

It suffices that the person who analyzes it adds a ‘new coordinate’ system to their minds, one that be proper for humans, not only for machines, and there is no paradox in The Liar.
If one insists in calling The Liar paradox, they can only be thinking of a paradox of mind, what is not worth discussing, for the mind is not, and should not be, a logical place.

The mind is a messy place, where we really do not want to go when writing about Philosophy: We wish to work with what we may reach and the mind is obviously a place that we cannot reach with symbols only.

It is well beyond that.

Thus, paradox of internal nature, therefore no paradox at all, for it will be for some, but not for others, not deserving universal standing as a paradox.

Conclusion

The Liar is an allurement as much as The Sorites is. The Liar exists to show the complexity of the human minds while The Sorites exists to show the complexity of the human verbal expression.

Basically, it is an allurement, something to remind us of how our mind is complex, if we decide to include it in the body of Philosophy.

Place it belongs to? Philosophy of Mind.

The Liar can interest Philosophy exclusively under the light of Philosophy of Mind then. Notwithstanding, to place the problem under Philosophy, we need to correct it and to revise its wording, so that it is in agreement with the well-posedness theory for philosophical problems.
Bibliography


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