From Marginal Utility to Revealed Preference: Rational Reconstruction

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Abstract

This paper provides an exposition of the standard model of economic choice, applying Popper’s method of rational reconstruction. It emphasizes that the explanation of downward-sloping demand curve, which is the original explicandum of this model, does not entail explanation of choice as such. Two alternative variants of the model of are considered: one based on the notion of diminishing marginal utility and the other on the concept of ordinal utility. Each variant is understood as a tentative solution to a theoretical problem and clarifies why the former is replaced by the latter. It also shows how the latter relates to consumer preferences and choices and why choices remain unexplained by the model.

Keywords: rational reconstruction, utility, marginal utility, preference, choice, demand function, consumer theory
1 Introduction

This workshop’s goal is to summarize the state of knowledge in the field of will, action and decision. Economics is one of the disciplines called upon to contribute to this goal. Needless to say, the model of economic choice is the dominant model of choice used in the social sciences (usually under the label ‘rational choice model’). In spite of the fact that this model was already established in the first half of the 20th century, it is still subject to misunderstandings. In particular, it is often believed that the model of economic choice is a theory of choice, i.e. that its purpose is to explain choice (e.g. Wong 2006; see also references there). In fact, as I attempt to demonstrate in this paper, that model is incapable of explaining choice: it is rather a set of assumptions about choice, the original purpose of which is to explain observed market regularity – the downward-sloping demand curve. To achieve its aim, this paper applies the method of ‘rational reconstruction’, which is characterized in the next section.

2 Rational reconstruction

The method of ‘rational reconstruction’ was described by Popper (1947; 1957; 1979),1 and further discussed and elaborated by Agassi (1963) and Lakatos (1963a; 1963b; 1970). It can be portrayed by a scheme such as the one in Fig. 1.2

The starting point is a problem situation \( P_0 \): it may be an unexplained phenomenon, internal inconsistency of a theory, a clash between a theory and a factual statement, a theory and a methodological maxim or between two theories. To this problem situation, one or several tentative solutions, \( S_1, \ldots, S_n \), are suggested. These tentative solutions in turn give rise to new problem situations, \( P_1, \ldots, P_n \). In the process, several tentative solutions may be eliminated as inadequate.

I assume the following simple criterion for elimination: let \( P_i \) denote a set of problems solved by \( S_i \) and let \( P_j \) be the set of problems solved by \( S_j \). Thus \( S_i \) is eliminated if \( P_i \subsetneq P_j \). For example, in Fig. 1, \( S_2 \) would be superseded by \( S_1 \), because \( S_2 \) is able to solve only the problem \( P_0 \), while \( S_1 \) solves both \( P_0 \) and \( P_2 \).

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1 However, Popper was not the first to apply it. For instance, Adam Smith’s Essay on the History of Astronomy (Smith 1982) can be considered rational reconstruction in action. The method usually associated with Smith – the ‘conjectural history’ (Pavlík 2005) – can be understood as a version of rational reconstruction, or ‘internal history’, to use Lakatos’s (1970) term.

2 For similar schemes, see Popper (1979, 119, 243, 297).
Together with Popper, I assume that problem situations exist objectively (to use Popper’s language, they belong to ‘World 3’). This implies that scientists who made their discoveries need not be aware what problem they actually solved at the time of their discovery\(^3\). Thus, I am only interested in problem situations and solutions to them and not the economists who came up with these solutions. To quote Schumpeter (1954, 363n), “theorems and not persons are the heroes of our story” (See also Lakatos 1970, 127n). To strictly separate the rationally reconstructed ‘logical history’ from the ‘history of discoveries’, rather than referring to the main text, I relegate all references regarding the origins of ideas into the section titled ‘Guide to the literature’. This section not only matches theories and people, but also mentions some secondary literature.

So far as the topic of this paper is concerned, the method of rational reconstruction has been used several times before. Two of these works stand out. The first is Hicks’s book, *Value and the Capital* (Hicks 1978), which made important contributions to the model of economic choice. Hicks seem to use this method implicitly and perhaps intuitively, because there is no evidence that he was influenced by Popper, or even that he was interested in methodological issues at the time of writing the book.\(^4\)

The second work applying rational reconstruction to economic choice is Wong’s book, *Foundations of Paul Samuelson’s Revealed Preference Theory* (Wong 2006). According to his own words, Wong was the first to consciously use rational reconstruction in economics. There is, however, a difference between Wong’s version of the method and the version used in this paper. Wong works with problem situations as seen by *theorists* and not as seen by *him*, thus making use of knowledge the authors did not have when working on their problems. Wong’s version of rational reconstruction is thus ‘psychological’: although he pays lip service to

\(^3\) Popper (1979, 246) gives, among others, the example of Schrödinger, whose famous equation solved a problem discovered only later by Max Born.

\(^4\) According to Simkin (2001), Hicks and Popper probably never met. He also recollects that Hicks “did not become seriously interested in scientific methods until 1974” (Simkin 2001, 236) and, even after that year, his methodological views cannot be characterized as Popperian.
Popper’s concept of ‘World 3’, he is partly concerned with the ‘World 2’ – the aims, beliefs and constraints of the economists, i.e. with knowledge in the subjective sense.

In this paper, I consider two models of economic choice: one is built around the notion of diminishing marginal utility, while the other ignores this assumption and is formulated in terms of ordinal utility or, alternatively, in terms of preferences or choices. The latter is shown to replace the former by the criterion for elimination introduced above.

The rest of the paper proceeds as follows: Section 3 formulates the problem which the model of economic choice aims to solve, i.e. to explain why and under what conditions demand curve slopes downward. A tentative solution based on diminishing marginal utility is presented. In Section 4, the problems with marginal-utility-based solutions are exposed and a new tentative solution, based on the concept of ordinal utility, is expounded. Section 5 relates utility to individual preferences, while Section 6 provides the link between preferences and choices. Section 7 provides a summary and concluding remarks. Section 8 furnishes a guide to the literature.

3 Marginal utility theory

Classical economists take it as well-established fact that if a price of a good goes up, an individual will purchase less of that good and vice versa. Although they admitted the presence of a wealth-seeking ‘economic man’ lurking behind this fact, they had no theory for how individual choice translates into demand and, in fact, such a theory was beyond their main focus. From the end of the 19th century, economists became increasingly unsatisfied with merely postulating the fact and began to ask the question of why such regularity is observed. More precisely, the problem these economists attempted to solve can be formulated as follows:

(P1) It seems to be an empirical fact (supported also by introspection) that an increase in the price of a good results, other things remaining equal, in a decrease in its demanded quantity. Two questions arise:

(P1.1) What must be assumed about individual choice to obtain this result?

(P1.2) What is the precise meaning of ‘other things remaining equal’?

A possible answer to the (P1.1) states that individuals have utility functions with certain properties and choose, from among feasible alternatives, the one which

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5 Those familiar with standard microeconomics may want to skim through the sections 3-6, paying attention only to the text in italics, i.e. problems, assumptions, solutions and definitions.
provides the highest utility. The ‘other things’ that must be kept equal in response to (P1.2) are the prices of other goods and the marginal utility of money. This solution to (P1) will now be discussed in more detail.

We first introduce some notation: let \( X = \mathbb{R}^2_+ \) be a set of all bundles of goods. For simplicity, we assume throughout that bundles contain only two goods, 1 and 2, and we adopt a convention that our interest is centered on demand for good 1. Let \( p \in \mathbb{R}^2_+ \) denote a vector of exogenously given prices of the two goods, and \( Y \) is the exogenously given amount of money at the consumer’s disposal. We then define a budget set, \( B = \{ x \in X : \sum p_x \leq Y \} \), which is a set of all feasible \( x \) for a given \( p \) and \( Y \). The marginal utility of money is denoted as \( \lambda \). Finally, \( u = u(x) \) is a utility function, with \( u_i \) and \( u_{ii} \) respectively denoting first and second partial derivatives with respect to \( i \), where \( i = 1, 2 \).

A consumer’s optimization problem can be then written as follows:

\[
\max_{x} u(x) \quad \text{s.t.} \quad x \in B
\]

First, note that the consumer is constrained by his budget set. This means that, even if there are goods outside the budget set that are physically available, the consumer does not consider them as choice alternatives. In other words, it is assumed that the consumer respects both property rights and contracts, which we formulate as an explicit assumption of the model:

(A1) *Property rights and contracts are respected.*

Next, we make the assumption that the consumer not only solves the problem (1), but also that this solution is his actual choice.

(A2) *Consumer chooses the bundle \( x^* \), which solves (1).*

We now focus on the solution of the optimization problem (1). This solution is written as:
\[ \frac{u_1}{p_1} = \frac{u_2}{p_2} = \lambda \]  

(1.1)

\[ \sum p_x^* = Y \]  

(1.2)

The demand for the good 1, \( x_1(p,Y) \), is then obtained as a solution of this system of equations. Now, in order to solve (P1), it must be ensured that as \( p_1 \) goes down (up), \( x_1 \) goes up (down). This can be ensured by simultaneously assuming that \( \lambda \) (the marginal utility of money) is constant and the marginal utility of the good 1, \( u_1 \), decreases, as \( x_1 \) increases (i.e. \( u_{11} < 0 \)). Given these two assumptions, it is obvious from (1.1) that if e.g. \( p_1 \) goes down, \( u_1 \) must go down in the same proportion in order to keep \( \lambda \) constant. By the assumption of diminishing marginal utility, \( u_1 \) goes up if \( x_1 \) increases. We conclude that a decrease in \( p_1 \) results in an increase in \( x_1 \). Focusing on good 2, it is assumed that \( p_2 \) is constant and – again to keep \( \lambda \) constant – \( u_2 \) is independent of \( x_1 \).

We now list all the necessary assumptions:

(A3) \( Y \) is held constant.

(A4) \( p_2 \) is held constant.

(A5) \( u_i > 0 \) for \( i = 1, 2 \).

(A6) \( u(x) \) is stable over time.

(A7) Diminishing marginal utility. For \( i = 1, 2 \), \( u_{ii} < 0 \).

(A8) \( \lambda \) is held constant.

(A9) For \( i, j = 1, 2 \) and \( i \neq j \), \( u_{ij} = 0 \)

The first tentative solution to (P1) runs as follows:

(S1) In order to obtain a downward-sloping demand curve for a good \( x_1 \), the assumptions (A1) – (A9) must be satisfied.\(^6\)

\(^6\) Proofs to the claims made in “solutions” throughout this paper can be found in advanced microeconomic textbooks. The standard reference is Mas Colell et al. (1995).
4  **Ordinal utility theory**

The (S1) solution indeed solves the (P1) problem. Nonetheless, it gives rise to several new problems:

(P2) *Diminishing marginal utility is incompatible with ordinal concept of utility.*

(P3) *The assumption of the constant marginal utility of money is often implausible.*

(P4) *The marginal utility of a good (i) often depends on the amount of a good (j).*

(P5) *What must be assumed about consumer ‘desire’ so that a utility function representing these ‘desires’ can be constructed at all?*

The first two problems deserve comments.

Economists, in the late 19th and early 20th century, made an effort to clarify that, when they talked about ‘utility’, they had no psychological magnitude in mind; they thought of utility as a theoretical construct that accounted for the fact that something was desired by the consumer. They also suggested alternative names for the concept (such as ‘desirability, desiredness, wantability’, etc.) in order to avoid confusion with psychological or normative connotations. The problem was that this notion of non-psychological utility turned out to be incompatible with the assumption of diminishing marginal utility. Here is why: assume three commodity bundles $x'$, $x''$, and $x'''$. The assumption of diminishing marginal utility can be formulated only if consumer is able to compare not only $u(x')$, $u(x'')$, and $u(x''')$ and also differences $u(x') - u(x'')$, $u(x'') - u(x''')$, and $u(x''') - u(x''''')$ for any three bundles $x'$, $x''$, and $x'''$. In other words, the consumer must be able to evaluate the strength of his ‘desires’. This seems to imply a psychological rather than a purely formal concept of utility.

To express the same thing from a different point of view, note that if a utility function only represents consumer’s ‘desires’ and not their strength, then this utility function is not unique and many other functions are capable of doing the same job. Formally, if $u(x)$ represents consumer ‘desires’, then any $v(x) = f(u(x))$ with $f' > 0$, also represents these ‘desires’. We say that the utility function is ordinal, i.e. unique up to positive monotonic transformations. According to (S1), marginal utility diminishes, i.e. $u_{ii} < 0$. However, a function $v(x)$, also representing the same desires, need not necessarily have diminishing their marginal utility: $v_{ii} = f'u_{ii} + f''u_i$ always has the same sign as $u_{ii}$, only if $f'' = 0$. The following is an example:
(E1) Consider $u = x_1^{1/2}x_2^{1/2}$. Marginal utility is $u_i = x_2^{1/2}/2x_1^{1/2}$ and is decreasing be $u_{ii} = -x_2^{1/2}/4x_1^{1/2}$. Now let $v = u^4 = x_1^4x_2^2$. $v$ represents the same desires as $u$, because $v' = 4u^3 > 0$. However, marginal utility is increasing because $v_{ii} = 2x_2^3 > 0$.

As for (P3), the assumption of a constant marginal utility of money means that each additional dollar is as important for the consumer as the previous one. This may be an unproblematic assumption for cases where we deal with small amounts of money relative to total wealth. But this assumption is probably not plausible in the rest of the cases. Moreover, the solution (S1) makes no predictions as to what will happen when the assumption of the constant marginal utility of money is violated.

Now, the question arises, whether there is a solution to (P1) – (P5). The answer is “Yes”, as will be now demonstrated. First we show how the (P1) – (P4) problems are dealt with.

Consider the solution to the consumer optimization problem described by (1.1) and (1.2). Rearranging the equation (1.1) we get:

$$u_1 / u_2 = p_1 / p_2 = \lambda$$

It is now easy to show that the ratio $u_1 / u_2$ is the same for all utility functions representing the same ‘desires’, i.e. it is invariant under positive monotonic transformation: let $v(x) = f(u(x))$ with $f' > 0$; then $v_1 / v_2 = f' u_1 / f' u_2 = u_1 / u_2$. The ratio is called ‘marginal rate of substitution’ and is denoted by $MRS_{21}$.

(E2) Consider the utility functions $u = x_1^{1/2}x_2^{1/2}$ and $v = u^4 = x_1^4x_2^2$ from the example (E2). It is straightforward to show that marginal rates of substitutions derived from them are equal at each point: $u_1 / u_2 = x_2 / x_1 = v_1 / v_2$.

The problem (P2) is now solved and the assumption of diminishing marginal utility is no longer necessary. This might be felt as a loss, because diminishing marginal utility has a strong intuitive appeal. There are two answers to this: first, it is not contended that marginal utility is not in some sense diminishing – it may or may not be. What is claimed here is that this assumption is not required to solve problem (P1). Second, diminishing marginal utility is substituted with no less intuitive assumption of diminishing marginal rates of substitution, i.e. $dMRS_{21}/dx_1 < 0$. The
marginal rate of substitution is the rate at which a consumer is willing to substitute the good 2 with the good 1, while maintaining the same level of utility. Now, as \( x_1 \) goes up, the each additional unit of \( x_1 \) is less valued by the consumer, i.e. the less \( x_2 \) is he willing to sacrifice. In other words, MRS\(_{21}\) goes down.

The diminishing marginal rate of substitution thus represents the same idea as diminishing marginal utility. But the two are not identical, as can be seen from the following expression:

\[
\frac{dMRS_{21}}{dx_1} = \frac{d(u_1 / u_2)}{dx_1} = \frac{u^2_2 u_{11} - 2 u_{12} u_{11} + u^2_1 u_{22}}{u^2_1}
\]

Consider the assumption of (S1), i.e. \( u_1, u_2 > 0 \) and \( u_{11}, u_{22} < 0 \). Then, we can still obtain an increasing marginal rate of substitution if \( u_{ij} \) is negative and large enough, as illustrated with the next example:

\( (E3) \) Let us state the following utility function: \( u = 100x_1^{1/2} + 100x_2^{1/2} + x_1^5 + x_2^5 - 555x_1 x_2^6 \). Consider the contour passing through the point (0.7, 0.68). Marginal utilities for both goods are positive and diminishing at this point: \( u_1 \geq 5.6, u_2 \geq 4.7, u_{11} \leq -424.8 \) and \( u_{22} \leq -452.3 \). Yet MRS\(_{21}\) is increasing in the surroundings of this point: \( dMRS_{21} / dx_1 \geq 20.5 \).

We now focus on the solution of (P1) under the assumption of ordinal utility. From (1.1*) we find that if \( p_2 \) is constant and \( p_1 \) decreases, then if utility is also held constant, MRS\(_{21}\) must decrease as well. Decreasing MRS\(_{21}\) corresponds to an increase in \( x_1 \) (by the assumption of diminishing marginal rates of substitution); hence, we get downward sloping-demand curves, when all other prices and utility are held constant.

Note that we need not assume (A9), which solves the problem (P4). This demand function is different from the one obtained in (S1): they differ in which variables are held constant as the price of a good changes. To distinguish this demand function from the demand function obtained in (S1), we call it Hicksian demand. The demand function obtained in (S1) is called Marshallian demand.\(^8\)

\(^7\) Likewise, it is not difficult to show that the marginal rate of substitution can decrease even if the marginal utilities do not diminish. The assumption of diminishing marginal rates of substitution corresponds to the assumption of diminishing marginal utility, only if the goods in question are independent, i.e. if \( u_{ij} = 0 \).

\(^8\) It is also sometimes called Walrasian demand. See Mas-Colell et al. (1995).
completeness, we mention that Hicksian demands $h_i(p,u)$ can be obtained as the solution to the following optimization problem:

$$\min_x px \text{ s.t. } u \leq u(x)$$

As a last step, we examine some of the relationships between the Marshallian and Hicksian demands that will help address the problem (P3). First, observe that dropping the constant marginal utility of money assumption fails to influence the result that Hicksian demand functions decrease prices. Second, from (1.1) it can be seen that if the marginal utility of money is not constant, Marshallian demand functions may or may not decrease prices. The precise condition, determining how Marshallian demand changes with the change in price, is given by the Slutsky equation:

$$\frac{\partial x_i}{\partial p_1} = \frac{\partial h_i}{\partial p_1} - x_i \frac{\partial x_i}{\partial Y}$$

Marshallian demand for $i$ always decreases in $p_1$ if the second term in the equation (‘income effect’) is positive or if it is negative and sufficiently small.\(^9\) Formally:

$$x_i \frac{\partial x_i}{\partial Y} > 0 \text{ or } x_i \frac{\partial x_i}{\partial Y} < 0 \text{ and } \left| x_i \frac{\partial x_i}{\partial Y} \right| < \left| \frac{\partial h_i}{\partial p_1} \right|$$

If $\frac{\partial x_i}{\partial Y} = 0$, Hicksian and Marshallian demands are identical. This is shown in the next example.

(E4) Consider the quasi-linear utility function: $u(x_1,x_2) = f(x_1) + x_2$ where $f' > 0, f'' < 0$. Condition (1.1) then becomes:

$$\frac{f_1}{p_1} = \frac{1}{p_2} = \lambda$$

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\(^9\)This requirement excludes the so-called Giffen behavior.
If \( p_2 \) is held constant, \( \lambda \) must also be constant as in (S1). The demand for the good 1 then decreases with \( p_1 \) and is then independent of income and utility:

\[
x_1(p, B) = h(p, u) = g(p_2 / p_1)
\]

I now summarize new assumptions introduced in this section and then formulate a solution to the problems (P1) – (P4):

(A10) Diminishing marginal rate of substitution. \( dMRS_{12} / dx_1 < 0 \).

(A11) Utility is held constant.

(S2.1) Two types of demand functions are distinguished: Marshallian and Hicksian. In order to obtain a downward-sloping Hicksian demand curve for a good \( x_i \), the following must be assumed: (A1), (A4) – (A6) and (A10) – (A11). In order to obtain a downward-sloping Marshallian demand curve for a good \( x_i \), the condition (5) must satisfied and also the following must be assumed: (A1) – (A6) and (A10).

If we compare the conditions for Marshallian demand in (S1) and (S2), we find two differences: the first is that diminishing marginal utility (A7) is replaced by the assumption of diminishing marginal rates of substitution (A10), which also makes it possible to drop the assumption of goods independence (A9); the second is that the assumption of constant marginal utility of money (A8) is replaced by the condition (5).

It remains to address the problem (P5). This will be done in the next section.

5 Utility and preferences

Rather than talking about ‘desires’, economists prefer to work with the term ‘preferences’. It will be now shown, in response to (P5), how one can get from preferences to a utility function.

Instead of assuming a utility function, we are now going to assume that people are able to compare various bundles of goods according to their preferences. More formally, let \( \succeq \) (weak preference) be a binary relation on \( X \). For convenience in notation, two definitions are introduced:

(D1) Strict preference. \( x' \succ x'' \) if and only if \( x' \succeq x'' \) and not \( x'' \succeq x' \).

(D2) Indifference. \( x' \equiv x'' \) if and only if \( x' \succeq x'' \) and \( x'' \succeq x' \).
In order to obtain a utility function with the desired properties (as those in (S2)),
certain assumptions related to the preference relation must be introduced. These
assumptions are as follows:

(A12) Completeness. For any \( x', x'' \in X \), either \( x' \succeq x'' \), or \( x'' \succeq x' \), or both.

(A13) Transitivity. For any \( x', x'', x''' \in X \), if \( x' \succeq x'' \) and \( x'' \succeq x''' \) implies \( x' \succeq x''' \).

(A14) Continuity. Let \( B(x') = \{ x \in X : x \succeq x' \} \) and \( W(x') = \{ x \in X : x' \succeq x \} \). Then for all \( x', B(x') \) and \( W(x') \) are closed.

(A15) Strict monotonicity. For any \( x', x'' \in X \) such that \( x' \succeq x'' \), and \( x' \neq x'' \), \( x' \succ x'' \).

(A15*) Local insatiability. For every \( x' \in X \) and for every number \( \varepsilon > 0 \) there exist
\( x'' \in X \) such that \( |x' - x''| < \varepsilon \) for every \( i = 1, 2 \) and \( x'' \succ x' \).

(A16) Strict convexity. For any \( x', x'' \in X \) such that \( x' \succeq x'' \) and \( x' \neq x'' \), and for every \( k \in (0,1) \), \( kx' + (1-k)x'' \succ x'' \).

(A17) Preferences are stable over time.

Under (A12) – (A14) a utility function can be constructed. This function is ordinal,
i.e. it is unique up to positive monotonic transformations (as assumed in (S2)). (A15)
ensures that \( u_i > 0 \). In fact, a weaker assumption than (A15), namely (A15*) is often
sufficient for the purposes of demand theory, but this assumption is not easily
translatable into properties of utility function. We can see that working with utility
functions rather than preferences, forces us to be more restrictive than is actually
required. If a utility function is differentiable, then (A16) ensures that \( dMRTS_{21}/dx_1 < 0 \).
Finally, (A17) is an analogy to (A6).

We now summarize the solution to (P5).

(S2.2) Assume a weak preference relation defined on \( X \), satisfying the assumptions (A12) –
(A17); then utility function \( u(x) \) exists and has the required properties specified by (S2).

Apart from obtaining the solution to (P5), the solution (S2) can be reformulated by
dropping some of the assumptions necessary only for the construction the utility
function. Nonetheless, we will not attempt the reformulation here.

(S2.3) Assume a weak preference relation defined on \( X \), satisfying the assumptions (A12),
(A13) and (A15*) – (A17); then (P1) has a solution analogous to (S2.1).

Having now solved the problems (P1) – (P5), we will now examine testable (at
least in principle) predictions of the model specified by (S2.3). One ‘prediction’ is the
downward-slopping demand curve itself. But this prediction is not really a test of the
theory just presented, because the downward-slopping demand curve was an
explicandum, from which we originally started. Actually, our explicandum was more of a hunch than a well-established fact, since all the data necessary to construct an individual demand curve are usually not available. Therefore, we now must look more carefully into the implications of (S2.3).

6 Preferences and choices
It is necessary to explicitly relate consumer preferences to choices. This first requires a formalization of choice. The choice is typically modeled as a correspondence, attaching a non-empty subset $C(B)$ to each budget set $B$. $C(B)$ can be interpreted as a set of acceptable alternatives. Now, preferences are related to choices by the following assumption:

\begin{equation}
\text{(A18) For every } B' \text{ and every } x' \in C(B'), \text{ there is no } x'' \in B' \text{ such that } x'' \succ x'.
\end{equation}

The (A18) assumption says that chosen alternative is always weakly preferred to anything else; hence, a consumer never choses an inferior (according to his preferences, that is) alternative. Note that (A18) is an analogy to (A2). We now introduce one more assumption:

\begin{equation}
\text{(A19) Generalized Axiom of Revealed Preference (GARP). For any finite } N \text{ bundles } x^n \in X, n = 1, \ldots, n, \text{ if } x^n \succeq x^n_{n-1} \text{ for } n = 1, \ldots, N - 1, \text{ then it is not the case that } x^N \succ x^1.
\end{equation}

We are now finally in a position to relate preferences and choices:

\begin{equation}
\text{(S2.4) Assume that (A17) and (A18) hold; then (A19) is satisfied if the consumer preferences satisfy assumptions (A12), (A13) and (A15*).}
\end{equation}

\begin{equation}
\text{(S2.4) is also important for the following reason: if one is uncomfortable assuming the existence of a preference scale in the mind of a consumer, one can abandon this assumption altogether. If consumer choices satisfy (A19), then he acts as if he were maximizing his preferences, but it need not be assumed that he actually does so. This is demonstrated by the following example:}
\end{equation}
(E5) A consumer buys only two goods, 1 and 2 and uses a simple rule to allocate his income: he always spends the fraction of his income \( a \) on 1 and \((1 - a)\) on 2. He behaves as if maximizing the utility function 
\[
 u = x_1^a x_2^{1-a}. 
\]

7 Summary
I now summarize the rational reconstruction of the model of economic choice, attempted in this paper, by a diagram in Fig. 3.

![Diagram of the model of economic choice]

Fig. 2 Rational reconstruction of the model of economic choice

The original problem (P1) of downward-sloping demand curve was solved by the marginal utility theory (S1), which however, gave rise to four other problems (P2) – (P5). All these problems (including the original, (P1)) were solved by the ordinal utility theory (S2.1), which can be expressed in terms of preferences (S2.2) and (S2.3) or choices (S2.4).

The ‘trinity’ of the (S2) model should be emphasized: this model can be described in one of the three ‘languages’, where one ‘language’ can usually be translated into the other. This is described in Fig. 4.

![Diagram of the economic trinity]

Fig. 3 ‘Economic trinity’
The (S2) theory itself, of course, gives rise to new problems that were not discussed in this paper. Although many of these problems remain to be solved, (S2) remains the most satisfactory explanation of (P1) so far available.

Finally, I address the goal of this paper, which was to demonstrate that the model of economic choice does not entail an explanation choice. First, observe that the explicandum of the model is, generally speaking, a change in behavior (i.e. the direction of a change of demanded quantity of a good) in response to a change in a constraint (caused by a change in price of that good). The aim is thus not to explain why a certain choice was made (i.e. particular quantity is chosen). One may get the false impression that choice is explained in the process of solving (P1); in particular, that choice is explained with preferences. Such an explanation would run as follows: ‘x was chosen because it was highest on the individual’s preference scale’. This however would not work because, as we have seen, this is true by assumption (A18). The concept of ‘choice’ and ‘preference’ are not independent of each other and therefore the former cannot be explained by latter. The inability of the model to explain choices becomes crystal clear if the model is formulated without reference to preferences or utility.

The question of whether the model of economic choice can contribute to the solution of the problems of “will, action and decision” (to reiterate the topic of this workshop) will be left open; nevertheless, I take the liberty of expressing my doubt that the answer is positive.

8 Guide to the literature

Section 2. For classical views on ‘economic man’, see e.g. Kirzner (1960), Machlup (1972), Oakley (1994), and Hudík (Forthcoming). (P1) was implicitly introduced by Marshall (1982) and is mentioned explicitly by Viner (1925), Hicks (1978), Samuelson (1974) and Houthakker (1961). (S1) was formulated by Marshall (1982).

Section 3. Problems (P2)-(P4) were identified and solved by (S2.1) by Hicks and Allen (1934a; 1934b) and Hicks (1978). Pieces of (S2) are already present in the works of Edgeworth (1881), Pareto (1971) and Slutsky (1952).

Section 4. The preference-based approach was formalized by Arrow (1963). The relationship between preferences and utility was studied by Debreu (1959).

Section 5. The choice-based model was introduced by Samuelson (1938; 1974). The formalization of choice is due to Arrow (1963). (S2.4) is due to Varian (1982), who built upon the contributions of Houthakker (1950) and Afriat (1967).
9 References


