

The Colour-Exclusion Problem

An Essay on the Problem of Colour-Exclusion in
Wittgenstein's Tractatus Logico-Philosophicus

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Index of contents

Abstract	3
1 Overview	3
1.1 Context	3
1.2 Review.....	5
2 Difficulties.....	6
3 Colour-Exclusion.....	8
4 Consequences	10
Glossary	12
Literature	13
Appendix	13

Abstract

There has been a long discussion about the seemingly large gap between Wittgenstein's early work, the *Tractatus Logico Philosophicus* (hereafter only TLP) and his posthumously published *Philosophical Investigations* (hereafter only PI). In this paper I argue that one of the main reasons for Wittgenstein turning away from the view uttered in the TLP was the Colour-Exclusion problem. I try to make clear what exactly the Colour-Exclusion problem is with regard to Wittgenstein's TLP and why it is so important.

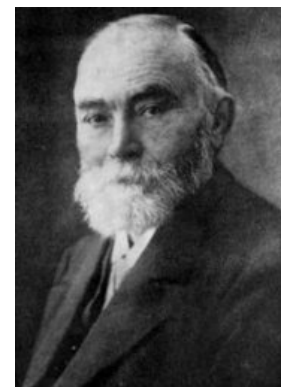
The first chapter makes clear what the purpose of the *Tractatus* is. I point out what Wittgenstein wanted to achieve and give a brief overview of the main theses. The second chapter will focus on the problems of the TLP and serves as a connection to the discussion of the Colour-Exclusion problem in the third chapter. The fourth and last chapter then delivers an insight into the influence the problem had on Wittgenstein's philosophy and methodology.

1 Overview

1.1 Context

The *Tractatus* can only be properly understood when set against the philosophy of Frege and Russell. I will therefore start by giving a short overview of their philosophical endeavours as far as they concern Wittgenstein's *Tractatus* and the Colour-Exclusion problem.

Gottlob Frege, a German mathematician, logician and philosopher, is usually regarded as one of the founders of analytical philosophy and can be credited with the invention of modern logic. Whereas the logic of Aristotle, which had changed little up to Frege, was based on the subject-predicate form, Frege introduced concepts and objects, allowing for far more flexibility.¹



¹ Egan, David. *SparkNote on Tractatus Logico-philosophicus*. 30 Oct. 2005
<<http://www.sparknotes.com/philosophy/tractatus/context.html>>

In greater detail: At the core of Aristotle's logic is the syllogism, the most famous of which perhaps being the following: "All men are mortal. Socrates is a man. Therefore, Socrates is mortal." which follows the general form "All A is B. Some C is A. Therefore, some C is B". In contrast to this, Frege's logic makes use of existential ("There exists an x") and universal ("For all x") quantifiers, so that "All men are mortal" can be written as "For all x in the class 'men', x is mortal", whereas "Socrates is mortal" would be expressed as "There exists an x in the class 'men' such that x is mortal and x is Socrates."²



I also find it important to make clear the difference between Kant's and Frege's use of the words "analytic" and "synthetic" as it shows the tremendous role Frege ascribed to logic and logical structures in general. »Kant [...] distinguished analytic and synthetic judgments according to how these judgments were framed in the mind. Frege insisted the analytic/synthetic distinction had nothing to do with psychology, but rather with justification: a judgment that can be justified by means of logic alone is analytic, whereas a judgment that must be justified by referring to the world is synthetic. Effectively, Frege argued that the meaning of sentences has nothing to do with what goes on in the head, and everything to do with their logical structure.«³ Wittgenstein inherited much of Frege's thought and his understanding of "analytic judgement" is much alike.



Inspired by Frege, Bertrand Russell together with Alfred North Whitehead wrote the *Principia Mathematica*, which was an effort to derive all of mathematics from

² Unkown author. *Aristotelian Logic*. In *BookRags*. 30 Oct. 2005
<<http://www.bookrags.com/sciences/mathematics/aristotelian-logic-wom.html>>

³ Egan, David. *SparkNote on Tractatus Logico-philosophicus*. 30 Oct. 2005
<<http://www.sparknotes.com/philosophy/tractatus/context.html>>

logical axioms. Both Frege and Russell saw logic as the most fundamental set of laws, which are universally applicable.

1.2 Review

Wittgenstein worked closely with Russell to whom he was introduced by Frege and shared the conviction that logic was a promising instrument for a complete understanding of the world. In his TLP, Wittgenstein tried to clarify what a logically constructed language can (and cannot) be used to say. Its main proposition is that language and reality share a common logical structure.

In greater detail: »On Wittgenstein's view, the world consists entirely of facts. (Tractatus 1.1) Human beings are aware of the facts by virtue of our [...] thoughts, which are most fruitfully understood as *picturing* the way things are. (Tractatus 2.1) These thoughts are, in turn, expressed in propositions, whose form indicates the position of these facts within the nature of reality as a whole and whose content presents the truth-conditions under which they correspond to that reality. (Tractatus 4) Everything that is true—that is, all the facts that constitute the world—can in principle be expressed by atomic sentences. Imagine a comprehensive list of all the true sentences. They would picture all of the facts there are, and this would be an adequate representation of the world as a whole.«⁴



⁴ Kemerling, Garth. *Ludwig Wittgenstein: Analysis of Language*. 28 Nov. 2005
<<http://www.philosophypages.com/hy/6s.htm>>

Ian Proops⁵ summarizes the core statements of Wittgenstein's TLP as follows:

- Every proposition has a unique final analysis which reveals it to be a truth-function of elementary propositions asserting the existence of atomic states of affairs (Tractatus 3.25, 4.21, 4.221, 5);
- Elementary propositions are mutually independent — each one can be true or false independently of the truth or falsity of the others (Tractatus 4.211, 5.134);
- Elementary propositions are immediate combinations of semantically simple symbols or “names” (Tractatus 4.221);
- Names refer to items wholly devoid of complexity, so-called “objects” (Tractatus 2.02, 3.22);
- Atomic states of affairs are combinations of these simple objects (Tractatus 2.01).

Now, since propositions "merely" express facts about the world they themselves do not convey any values. A logical language deals exclusively with what is true and false. It can not be used to make aesthetic or ethical judgements. What is beautiful or good can not be expressed, because it doesn't concern facts. It becomes clear that a satisfactory logical description of the way things are would eradicate all questions traditional philosophy is concerned with. (Tractatus 6.53)

2 Difficulties

As Garth Kemerling points out, »the problem with logical analysis is that it demands too much precision, both in the definition of words and in the representation of logical structure.«⁶ In ordinary language a word is often used in more than one specific meaning and a variety of grammatical forms may be used to express the same basic thought.

The main difficulty of logical analysis seems to be connected with finite and infinite case considerations in such a way as $(\exists x).fx$ doesn't correspond to any reality and therefore

⁵ Proops, Ian. *Wittgenstein's Logical Atomism*. In *The Stanford Encyclopedia of Philosophy* (Winter 2004 Edition). Edward N. Zalta (ed).
<<http://plato.stanford.edu/archives/win2004/entries/wittgenstein-atomism/>>

⁶ Kemerling, Garth. *Ludwig Wittgenstein: Analysis of Language*. 28 Nov. 2005
<<http://www.philosophypages.com/hy/6s.htm>>

can not be analysed. The logical product of elementary propositions is not the world but the logical product of elementary propositions. In 1932 Wittgenstein acknowledges that his main misconception in the TLP was confusing logical analysis with chemical analysis. »There is a most important mistake in [the] 'Tract[atus] ... I pretended that [a] proposition was a logical product; but it isn't, because '...' don't give you a logical product. It is [the] fallacy of thinking $1 + 1 + 1$ is a sum. It is muddling up a sum with the limit of a sum.«⁷ and »I thought $(\exists x).fx$ is a definite logical sum, only I can't at the moment tell you which. «⁸

The question now is this: What caused Wittgenstein to challenge his believes uttered in the TLP? Why did he reconsider his position after allegedly having solved all problems of philosophy? The answer seems to be the so-called Colour-Exclusion problem, a difficulty that arises from the Tractarian view that every elementary proposition can be true or false regardless of the truth or falsity of any other elementary proposition. »In view of its generality, the problem might more accurately be termed "the problem of the manifest incompatibility of apparently unanalysable statements."«⁹

»The problem may be stated as follows: Suppose that A is a point in the visual field. Consider the propositions P: "A is blue" and Q: "A is red" (supposing "red" and "blue" to refer to determinate shades). It is clear that P and Q cannot be true together; and yet, on the face of it, it seems that this incompatibility (or "exclusion" in Wittgenstein's parlance) is not a logical impossibility. In the Tractatus Wittgenstein's response was to

⁷ Wittgenstein, Ludwig. *No Title*. In *Cambridge University, G. E. Moore Archive, catalogue reference: ADD 8875, 10/7/7*. Cited by Ian Proops. In *The New Wittgenstein: a Critique*. 29 Nov. 2005 <<http://www-personal.umich.edu/~iproops/practice/nw.pdf>>

⁸ Wittgenstein, Ludwig. *No Title*. In *Cambridge University, G. E. Moore Archive, catalogue reference: ADD 8875, 10/7/7*. Cited by Ian Proops. In *The New Wittgenstein: a Critique*. 29 Nov. 2005 <<http://www-personal.umich.edu/~iproops/practice/nw.pdf>>

⁹ Proops, Ian. *Wittgenstein's Logical Atomism*. In *The Stanford Encyclopedia of Philosophy (Winter 2004 Edition)*. Edward N. Zalta (ed.). <<http://plato.stanford.edu/archives/win2004/entries/wittgenstein-atomism/>>

treat the problem as merely apparent. He supposed that in such cases further analysis would always succeed in revealing the incompatibility as logical in nature.«¹⁰

3 Colour-Exclusion

»For two colours, e.g., to be at one place in the visual field is impossible, and indeed logically impossible, for it is excluded by the logical structure of colour. / Let us consider how this contradiction presents itself in physics. Somewhat as follows: That a particle cannot at the same time have two velocities, that is, that at the same time it cannot be in two places, that is, that particles in different places at the same time cannot be identical. / It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The assertion that a point in the visual field has two different colours at the same time, is a contradiction.« (Tractatus 6.3751)

»As Frank Ramsey¹¹ observes in his review of the Tractatus, [Wittgenstein's] analysis [...] fails to reveal a logical incompatibility between the two statements in question; for, even granting the correctness of the envisaged reduction of the phenomenology of colour perception to facts about the velocities of particles, the fact that one and the same particle cannot be (wholly) in two places at the same time still looks very much like a synthetic a priori truth. It turns out, however, that Wittgenstein was well aware of this point. He knew that he had not taken the analysis far enough to bring out a logical contradiction, but he was confident that he had taken a step in the right direction. [...] [In a Notebooks entry from August 1916] Wittgenstein is conjecturing that it will turn out to be a conceptual (hence, for him logical) truth about particles and space (and presumably also time) that particles in two distinct places (at the same time) are distinct. He does not yet possess the requisite analyses to demonstrate this conjecture, but he is optimistic that they will be found.«¹²

¹⁰ Proops, Ian. *Wittgenstein's Logical Atomism*. In *The Stanford Encyclopedia of Philosophy* (Winter 2004 Edition). Edward N. Zalta (ed.). <<http://plato.stanford.edu/archives/win2004/entries/wittgenstein-atomism/>>

¹¹ Ramsey, Frank. *Mind*. 1923. vol. 32, no. 128, pages 465–74.

¹² Proops, Ian. *Wittgenstein's Logical Atomism*. In *The Stanford Encyclopedia of Philosophy* (Winter 2004 Edition). Edward N. Zalta (ed.). <<http://plato.stanford.edu/archives/win2004/entries/wittgenstein-atomism/>>

However, there is an essential and irreducible difference between the two propositions "A is red" + "A is blue" and "A is red" + "A is not red". While the latter is evidently a formal, i.e. logical contradiction, the former is not as we have to rely on the "logical system of colours" to understand that both cannot be true at the same time. Wittgenstein's essay *Some Remarks on Logical Form* owns up to this and marks the end of his optimism that a complete analysis will get rid of the problem of Colour-Exclusion. He arrives at the view that some incompatibilities cannot be reduced to logical impossibilities and as a consequence he abandons the idea of an all-embracing logical system and replaces it by allowing for numerous decentralized systems of sentences. From this follows that it is not a single proposition anymore that is laid against reality like a ruler (Tractatus 2.1512) but a system of propositions. By asserting that all the graduating lines of the ruler touch the object simultaneously, if an object reaches up to the third mark I can then infer that it doesn't reach to the fourth. Analogous, if I know "A is red" than I do also know that it is not blue, green or any other colour than red because I laid the entire colour scale against it. From this follows then that an elementary proposition's being true or false is not independent from the truth or falsity of other elementary propositions.

Yet another problem turns up in connection with colours, which is closely related to degrees of qualities in general (e.g. the shade of a colour, the pitch of a tone ...). As a certain shade of red can not be arrived at by logically adding up redness, numbers have to enter into elementary propositions. This is because if we try, for example, to analyse the following given statements "A has exactly two degrees of brightness" + "A has exactly one degree of brightness" to bring out the logical impossibility of their being true together, we face insurmountable difficulties. Wittgenstein believed the most plausible suggestion to be to »adapt the standard definitions of the numerically definite quantifiers to the system described in the Tractatus, analysing these claims as respectively: " $(\exists x) (B(x) \ \& \ A \text{ has } x) \ \& \ \sim(\exists x,y) (B(x) \ \& \ B(y) \ \& \ A \text{ has } x \text{ and } A \text{ has } y)$ " ("Bx" means "x is a degree of brightness") and " $(\exists x,y) (B(x) \ \& \ B(y) \ \& \ A \text{ has } x \text{ and } A \text{ has } y) \ \& \ \sim(\exists x,y,z) (B(x) \ \& \ B(y) \ \& \ B(z) \ \& \ A \text{ has } x \ \& \ A \text{ has } y \ \& \ A \text{ has } z).$ "¹³ But the trouble is, that the

¹³ Proops, Ian. *Wittgenstein's Logical Atomism*. In *The Stanford Encyclopedia of Philosophy* (Winter 2004 Edition). Edward N. Zalta (ed.).
<http://plato.stanford.edu/archives/win2004/entries/wittgenstein-atomism/>

analysis makes it seem as though brightness was some kind of corpuscle whose association with a thing made it bright¹⁴ and we could ask of an object with one degree of brightness which one of the two – x or y – it has. The impossibility of something having both exactly one and exactly two degrees of brightness therefore emerges as an irreducible mathematical impossibility.

Wittgenstein seems to have thought in the TLP that all quantities could be reduced to logical entities provided the analysis is advanced far enough. Now, as logic alone turns out to be unable to fully describe the complexity of physical reality, necessity can, after all, not be reduced to logical necessity, which contradicts Tractatus 6.37 and is a devastating conclusion for the philosophical enterprise of understanding the world by means of logic.

4 Consequences

Wittgenstein, however, didn't want to reject the project of the TLP as a whole yet, so instead he adapted his truth tables to accommodate for the fact of evident dependency among certain of them and let numbers enter into propositions to allow for degrees of qualities.

Only later did he realize that »developing an ideal formal language that accurately pictures the world is not only impossibly difficult but also wrong-headed. Since philosophical problems arise from the intellectual bewilderment induced by the misuse of language, the only way to resolve them is to use examples from ordinary language to deflate the pretensions of traditional thought. On this conception of the philosophical enterprise, the vagueness of ordinary usage is not a problem to be eliminated but rather the source of linguistic riches. It is misleading even to attempt to fix the meaning of particular expressions by linking them referentially to things in the world. The meaning of a word or phrase or proposition is nothing other than the set of (informal) rules governing the use of the expression in actual life.«¹⁵

¹⁴ Klagge, James & Nordmann Alfred. eds., *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*. 1993. Indianapolis and Cambridge.

¹⁵ Kemerling, Garth. *Ludwig Wittgenstein: Analysis of Language*. 28 Nov. 2005 <<http://www.philosophypages.com/hy/6s.htm>>

In conclusion I'd like to emphasize that Wittgenstein did not abandon the Tractarian view instantly but used it as basis for his philosophical work for many years. Although he pervasively revised the ideas of the TLP, their impact on his later thought is undeniable.

Also, it should be noted that the colour exclusion problem appears in two different variants in Wittgenstein's complete works. Firstly in the form discussed in this paper, i.e. that "A is red" + "A is blue" interferes with the independency of elementary propositions. The problem is close to the recognition that a particle can't move at two different velocities at a given time. It is strongly bound to the TLP. Secondly it appears particularly in Wittgenstein's *Remarks on Colour* as the question whether from "A is red" + "A is blue" follows that "A is violet". Similarly, does a colour like greenish red exist? The second variant is concerned with the colour octahedron and the problem of mixture of colours. Close to this is also the problem of transparent white. From this alone we can easily see that Wittgenstein was concerned with colours from start to finish of his philosophical enterprise and closer investigation reveals that the colour theme runs through Wittgenstein's works like a thread. Accordingly, the importance of the subject can hardly be overestimated.

Glossary

Analytic Philosophy - Analytic philosophy is primarily defined by its approach. Precise argumentation and clear evidence are of predominant importance and much weight is being placed on avoidance of ambiguity and attention to detail. It is the dominant philosophical method of English-speaking countries.

Fact - A fact is a complex made up of states of affairs. It can either be the case or not be the case. The world is the totality of existent, i.e. positive facts. (Tractatus 2.034 + 2.04 + 2.06)

Thought - Thought, in the Tractarian sense, refers to a logical entity, which shares the logical form of facts and propositions. Based on this commonality we are able to put our thoughts into the world in the form of propositions that picture reality. (Tractatus 3.01 + 3.03)

Picture theory - The sense of a proposition does not need to be made clear by means of elucidations. A proposition and the reality it depicts share a logical form. Thus, there is nothing external to the proposition that can make the connection any clearer than it already is. (Tractatus 4.01 + 4.021)

Proposition - A proposition is the means of expressing a thought. It is an articulate description of the constituents of a complex configuration. A proposition can only say how things are, not what they are. It employs simple signs to refer to objects and serves as a picture of the facts it represents. The simplest form of a proposition is called elementary proposition. (Tractatus 3.1 + 3.141 + 3.21 + 3.221)

State of affairs - A state of affair is a combination of objects. States of affairs are utterly simple, unanalyzable, and mutually independent. The totality of states of affairs is the world. (Tractatus 2.01 + 2.04 + 2.061)

Object - Objects are simple items that constitute states of affairs. Although they make up the substance of the world, they can only exist within the context of states of affairs. Objects have internal properties, i.e. their logical form, and external properties, i.e. whatever properties are ascribed to them in states of affairs. (Tractatus 2.01 + 2.013 + 2.01231 + 2.0141 + 2.02)

Contradiction - A proposition that is false no matter what is the case or is not the case. A contradiction lacks sense, but is not nonsensical. (Tractatus 5.101 + 5.142)

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Appendix

Table of Logical Notation

Logical Notion	Modern Notation
It is not the case that Fx	$\neg Fx$
If Fx then Gy	$Fx \rightarrow Gy$
Every x is such that Fx	$\forall x Fx$
Some x is such that Fx	$\neg \forall x \neg Fx$ or $\exists x Fx$
Every F is such that Fa	$\forall F Fa$
Some F is such that Fa	$\neg \forall F \neg Fa$ or $\exists F Fa$